ODE

Method → Qulitative Analyses → Slope Field / Phase Plane

Method → Quantitative Analyses → Exact Solution(*)

Quantitative Analyses → Approximation

(*) : The solution may be "<u>local</u>" :

By: $\varphi(x, c_1 \cdots c_n) = \varphi$ $\Rightarrow$ $\begin{cases} \varphi(x, c_1 \cdots c_n) = \varphi \\ \varphi^{(1)}(x, c_1 \cdots c_n) = \varphi^{(1)} \\ \vdots \\ \varphi^{(n-1)}(x, c_1 \cdots c_n) = \varphi^{(n-1)} \end{cases}$ If $\dfrac{D(\varphi, \cdots \varphi^{(n-1)})}{D(c_1 \cdots c_n)} = \left| \left(\dfrac{\partial \varphi^{(i)}}{\partial c_j} \right)_{n \times n} \right| \neq 0$

By Implicit Function Thm :

We can obtain: $\begin{cases} c_1 = c_1(x, \varphi, \cdots \varphi_{n-1}) \\ \vdots \\ c_n = c_n(x, \varphi, \cdots \varphi_{n-1}) \end{cases}$ where $\varphi_i = \varphi^{(i)}$ locally.

$\Rightarrow$ Then $\varphi(x, c_1, c_2 \cdots c_n)$ is the exact solution

of $\varphi^{(i)}(x, c_1(x, \varphi, \cdots \varphi_{n-1}), c_2(*) \cdots, c_n(*)) = \varphi^{(i)}$

$0 \le i \le n$